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Hawking radiation as tunnelling: the *D*-dimensional rotating case

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Abstract
The tunnelling method for Hawking radiation is revisited and applied to the *D*-dimensional rotating case. Emphasis is given to covariance of results. Certain ambiguities afflicting the procedure are resolved.

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1. Introduction

It is well known that black holes at classical level are black, namely nothing can escape from the event horizon; but if one takes into account the quantum effects on the matter, they are no longer black and the Hawking radiation is present. In particular, it has been shown that this radiation is directly connected with the leading short-distance singularity of the two-point function of the matter fields on the horizon [1]. Essentially this fact can also be seen (with much less rigour, though) in the tunnelling method to be discussed below.

It is also well known that there exist several derivations of Hawking radiation and recently there has been a renewed interest in this long standing issue in QFT in curved spacetime. In fact, Parikh and Wilczek [2] introduced a method involving the calculation of the classical action along classically forbidden trajectories, starting just behind the horizon and travelling outward to infinity. Thus, the particle must travel backward in time, and its classical action I becomes imaginary (signalling the impossibility of such motion), and this imaginary part can be viewed as a 'first quantization' transition amplitude. The imaginary part is governed by the short-distance behaviour on the horizon of the classical action. In their tunnelling approach, they claimed the relevance of Painlevé stationary gauge for 4-dimensional Schwarzschild BH

$$ds^2 = -\left(1 - \frac{2mG}{r}\right) dT^2 - 2m\, dr\, dT + dr^2 + r^2\, dS_2^2.$$

Instead, we will show how it is possible to work in the 4-dimensional Schwarzschild static gauge

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2MG}{r}\right)} + r^2 \, dS_2^2,$$

and will extend the discussion to D-dimensional rotating cases.

The derivation is quite elementary and is based on the computation of the classical action I along a trajectory starting from the horizon and ending in the bulk and the associated WKB approximation,

$$\text{amplitude} \propto e^{iI}. \tag{1.1}$$

The related semiclassical emission rate is, ignoring the prefactor,

$$\Gamma \propto |\text{amplitude}|^2 \propto e^{-2\,\text{Im}\, I},$$

where $\text{Im}\, I$ is the imaginary part of I. It will be shown that this semiclassical emission rate can be written as a Boltzmann statistical factor

$$\Gamma \propto e^{-\beta E},$$

where E is the energy of the particle, and β will be interpreted as the inverse of temperature of the emitted radiation.

2. The static case

A generic static D-dimensional BH metric ansatz in the Schwarzschild static gauge, $x^\mu = (t, r, x_i)$, reads

$$ds^2 = -A(r)\, dt^2 + \frac{dr^2}{B(r)} + C(r) h_{ij}\, dx^i\, dx^j = g_{\mu\nu}\, dx^\mu\, dx^\nu. \tag{2.1}$$

For black hole solutions, $A(r)$ and $B(r)$ vanish on the horizon $r = r_H$. Beside the WKB approximation, the other relevant ingredient is the classical action I of a particle in the above black hole background. One can evaluate it by making use of the relativistic Hamilton–Jacobi equation

$$g^{\mu\nu} \partial_\mu I \partial_\nu I + m^2 = 0. \tag{2.2}$$

Since the metric is static and spherical symmetric, one has

$$I = -Et + W(r) + J(x^i), \tag{2.3}$$

and for $W(r)$, the following expression is obtained:

$$\frac{dW}{dr} = \frac{1}{\sqrt{A(r)B(r)}} \sqrt{E^2 - A(r)\left(m^2 + \frac{J^2}{C(r)}\right)}. \tag{2.4}$$

The action along a path connecting the horizon and a point in the bulk exterior region is

$$I = -Et + \int_{r_H}^{r_1} \frac{dr}{\sqrt{A(r)B(r)}} \sqrt{E^2 - A(r)\left(m^2 + \frac{J^2}{C(r)}\right)} + J(x_i). \tag{2.5}$$

For non-extremal black holes, the integrand has a non-integrable singularity at the horizon which dominates the integral; splitting the integration into near-horizon and the bulk contributions, and considering only the near-horizon one, we have

$$A(r) = A'(r_H)(r - r_H) + \cdots, \qquad B(r) = B'(r_H)(r - r_H) + \cdots. \tag{2.6}$$

Thus, $\sqrt{A(r)B(r)} \propto (r - r_H)$. As a consequence, to define the integral we have to use a regularization. However, physical Feynman prescription, naively applied to the coordinate r, namely

$$\frac{1}{r - r_H} \to \frac{1}{r - r_H - \mathrm{i}0} = PP\frac{1}{r - r_H} + \mathrm{i}\pi\delta(r - r_H) \tag{2.7}$$

turns to be meaningless in curved spacetimes, due to the lack of covariance.

Let us illustrate this issue in $D = 4$ and for Schwarzschild black hole in the standard spherical coordinates (t, r, θ, ϕ). Here we have

$$A(r) = B(r) = 1 - \frac{r_H}{r}, \qquad C(r) = r^2,$$

where $r_H = 2MG$ is the horizon radius. The classical action reads

$$I = -Et + \int_{r_H}^{r_1} \frac{r\,\mathrm{d}r}{r - r_H}\sqrt{E^2 - \frac{r - r_H}{r}\left(m^2 + \frac{J^2}{r^2}\right)} + J(x_i). \tag{2.8}$$

The naive use of the above prescription leads to

$$I = \mathrm{i}\pi r_H E + \text{real part} = \mathrm{i}\pi 2MGE + \text{real part}. \tag{2.9}$$

For small E, we may, as a leading approximation, neglect the back reaction on the black hole geometry. Thus

$$\Gamma \propto \mathrm{e}^{-4\pi MGE}, \tag{2.10}$$

which turns out to be half of the correct result.

On the other hand, we can use isotropic spherical coordinates, $r \to \rho$, defined by

$$r = \rho\left(1 + \frac{\rho_H}{\rho}\right)^2, \qquad \rho_H = \frac{r_H}{4}. \tag{2.11}$$

As a result, the general metric we started from, reads with ρ instead of r

$$\mathrm{d}s^2 = -A(\rho)\,\mathrm{d}t^2 + \frac{1}{B(\rho)}\left(\mathrm{d}\rho^2 + \rho^2\,\mathrm{d}S_2^2\right),$$

where

$$A(\rho) = \left(\frac{\rho - \rho_H}{\rho}\right)^2\left(1 + \frac{\rho_H}{\rho}\right)^{-2},$$

$$B(\rho) = \left(1 + \frac{\rho_H}{\rho}\right)^{-4}, \qquad C(\rho) = \rho^2\left(1 + \frac{\rho_H}{\rho}\right)^4,$$

$$\sqrt{A(\rho)B(\rho)} = \frac{\rho - \rho_H}{\rho}\left(1 + \frac{\rho_H}{\rho}\right)^{-3}.$$

The naive use of the prescription in the previous formula leads to

$$I = \mathrm{i}\pi 8\rho_H E + \text{real part} = \mathrm{i}\pi 4MGE + \text{real part}, \tag{2.12}$$

namely the correct result

$$\Gamma \propto \mathrm{e}^{-8\pi MGE}, \tag{2.13}$$

since $T_H = \frac{1}{8\pi MG}$ is the Hawking temperature for the Schwarzschild black hole.

Why is this discrepancy present? Because the covariance is missing! It can be recovered by means of the proper spatial distance

$$\mathrm{d}\sigma^2 = \frac{\mathrm{d}r^2}{B(r)} + C(r)h_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j. \tag{2.14}$$

This is a crono-invariant quantity, namely invariant under the action of the group of time recalibration and spatial diffeomorphisms [3–5].

Limiting to the s-wave contribution (the bulk of particle emission is contained in such modes), one gets

$$\sigma = \int \frac{\mathrm{d}r}{\sqrt{B(r)}}, \tag{2.15}$$

and

$$W(\sigma) = \int \frac{\mathrm{d}\sigma}{\sqrt{A(r(\sigma))}} \sqrt{E^2 - A(r(\sigma))m^2}. \tag{2.16}$$

Splitting again the near-horizon and bulk contributions, in the so-called near-horizon approximation,

$$\sigma = \frac{2}{\sqrt{B'(r_H)}} \sqrt{r - r_H} + \cdots, \tag{2.17}$$

$$A(r(\sigma)) = A'(r_H)B'(r_H)\frac{\sigma^2}{4} + \cdots. \tag{2.18}$$

Thus, one gets the invariant result

$$W(\sigma) = \frac{2}{\sqrt{A'(r_H)B'(r_H)}} \int \frac{\mathrm{d}\sigma}{\sigma} \sqrt{E^2 - A(r(\sigma))m^2} + \cdots. \tag{2.19}$$

The function $1/\sigma$ restricted to positive σ can only be used to define a real distribution, with no imaginary part. Hence we must let it take a small negative value, the physical justification for this being that we are discussing an off-shell process, whereby the particle does not really travel in Lorentzian spacetime during the tunnelling process.

Making then use of the Feynman prescription

$$\frac{1}{\sigma} \to \frac{1}{\sigma - \mathrm{i}0} = PP\frac{1}{\sigma} + \mathrm{i}\pi\delta(\sigma), \tag{2.20}$$

it follows that I acquires an imaginary part

$$I = \frac{2\pi \mathrm{i}}{\sqrt{A'(r_H)B'(r_H)}} E + (\text{real contribution}). \tag{2.21}$$

As a result, in the leading approximation (small E and no back reaction), the semi-classical emission rate is given by the general formula [6]

$$\Gamma \equiv \mathrm{e}^{-2\,\mathrm{Im}\, I} = \mathrm{e}^{-\frac{4\pi E}{\sqrt{A'(r_H)B'(r_H)}}} = \mathrm{e}^{-\beta_H E}. \tag{2.22}$$

This is the standard Boltzmann factor as soon as one recognizes that

$$\beta_H = \frac{4\pi}{\sqrt{A'(r_H)B'(r_H)}} \tag{2.23}$$

represents the inverse Hawking temperature of the radiation. This result is in agreement with surface gravity and conical singularity methods.

The improved tunnelling method also correctly sets the temperature for the extremal black holes equal to zero [6].

In these cases, the integral of the radial part of the action turns out to be divergent, but with a pole of second order in $r - r_H$. This implies that the horizon is at infinite proper distance, and there is no analytic covariant regularization giving an imaginary part of the action. As a consequence, the Hawking temperature is vanishing. We stress that a naive use of the coordinate r in computing and regularizing the integrals leads to a non-vanishing, thus incorrect, temperature.

3. The rotating case

Our approach, with some reasonable approximation, may be extended to the rotating (stationary) case.

As a starting point, we consider a D-dimensional black hole solution in the Boyer–Lindquist coordinates $x^\mu = (t, r, x^a, x^A)$, x^a and x^A being angular coordinates. We assume that $g_{\mu\nu} = g_{\mu\nu}(r, x^A)$, and, as a consequence, $\partial_t g_{\mu\nu} = 0$ but also $\partial_a g_{\mu\nu} = 0$. The angular coordinate x^a is associated with the axial symmetries which are present and the only non-vanishing off-diagonal time–space components are g_{ta}. Thus, we have

$$\mathrm{d}s^2 = g_{tt}\,\mathrm{d}t^2 + 2g_{ta}\,\mathrm{d}t\,\mathrm{d}x^a + g_{rr}\,\mathrm{d}r^2 + g_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b + g_{AB}\,\mathrm{d}x^A\,\mathrm{d}x^B. \tag{3.1}$$

For our purpose, we are interested in the ADM-like form. It reads

$$\mathrm{d}s^2 = -N^2\,\mathrm{d}t^2 + g_{ab}(\mathrm{d}x^a + N^a\,\mathrm{d}t)(\mathrm{d}x^b + N^b\,\mathrm{d}t) + g_{rr}\,\mathrm{d}r^2 + g_{AB}\,\mathrm{d}x^A\,\mathrm{d}x^B, \tag{3.2}$$

where

$$A = -N^2 = g_{tt} - (g_{ab})^{-1} g_{ta} g_{tb}, \; N^a = -(g_{ab})^{-1} g_{ta} g_{tb}. \tag{3.3}$$

This form of the metric selects the two functions $A = -N^2$ and $B = g^{rr} = 1/g_{rr}$. These two functions are typically vanishing on the horizon and making use of the near-horizon approximation, one can formally deal with a situation similar to the static cases previously considered. At this point, one may recall that in the Kerr–Newman family of solutions in-(out-)going geodesics are perpendicular to the horizon, so the near-horizon approximation implies the selection of s-modes only, as far as the tunnelling method is concerned.

Let us illustrate this general procedure with some examples. The first non-trivial example is the $D = 4$ Kerr–AdS black hole. In Boyer–Lindquist coordinates it reads

$$\mathrm{d}s^2 = -\frac{\Delta_r}{\rho^2}\left[\mathrm{d}t - \frac{a\sin^2\theta}{\Xi}\,\mathrm{d}\phi\right]^2 + \frac{\rho^2}{\Delta_r}\,\mathrm{d}r^2 + \frac{\rho^2}{\Delta_\theta}\,\mathrm{d}\theta^2 + \frac{\Delta_\theta\sin^2\theta}{\rho^2}\left[a\,\mathrm{d}t - \frac{r^2+a^2}{\Xi}\,\mathrm{d}\phi\right]^2, \tag{3.4}$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta, \qquad \Xi = 1 - \frac{a^2}{l^2}, \tag{3.5}$$

$$\Delta_r = (r^2+a^2)\left(1 + \frac{r^2}{l^2}\right) - 2mr, \qquad \Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\theta. \tag{3.6}$$

Here a denotes the rotational angular momentum parameter and the negative cosmological constant is $\Lambda = -\frac{3}{l^2}$.

The ADM form, defining the lapse function $A(r,\theta)$ and $g^{rr} = g_{rr}^{-1} = B(r,\theta)$ can easily be computed and reads

$$\mathrm{d}s^2 = -A(r,\theta)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r,\theta)} + \frac{\rho^2}{\Delta_\theta}\,\mathrm{d}\theta^2 + \frac{\Sigma^2\sin^2\theta}{\rho^2\Xi^2}(\mathrm{d}\phi - \omega\,\mathrm{d}t)^2, \tag{3.7}$$

where

$$A(r,\theta) = \frac{\rho^2\Delta_\theta\Delta_r}{\Sigma^2}, \qquad B(r,\theta) = \frac{\Delta_r}{\rho^2}, \tag{3.8}$$

$$\Sigma^2 = \Delta_\theta(r^2+a^2)^2 - \Delta_r a^2\sin^2\theta, \tag{3.9}$$

$$\omega = \frac{a(\Delta_\theta(r^2+a^2) - \Delta_r)\Xi}{\Sigma^2}. \tag{3.10}$$

At the horizon, $g^{rr} = 0$, namely $\Delta_r(r_H) = 0$. Note the A and B, functions of the metric, have an explicit dependence on the angular coordinate θ.

The strategy can be described as follows: one has to expand the quantities A and B near the horizon only with respect to r

$$A(r,\theta) = A'(r_H,\theta)(r - r_H) + \cdots ' \tag{3.11}$$

$$B(r,\theta) = B'(r_H,\theta)(r - r_H) + \cdots . \tag{3.12}$$

As a result

$$\mathrm{d}s^2 = -A'(r_H,\theta)(r - r_H)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B'(r_H,\theta)(r - r_H)} + \frac{\rho^2(r_H,\theta)}{\Delta_\theta}\,\mathrm{d}\theta^2 + \frac{\Sigma^2(r_H,\theta)\sin^2\theta}{\rho^2(r_H,\theta)\,\Xi^2}\,\mathrm{d}\chi^2, \tag{3.13}$$

where

$$\chi = \phi - \Omega t, \qquad \Omega = \omega(r_H). \tag{3.14}$$

We also may consider the trajectories with θ and χ constants. Actually, it is a well-known property of the Kerr black hole, shared by the KAdS solution, that along geodesics in surfaces $\theta = \theta_0$ constant, the combination $\phi - \Omega t$ is finite on the horizon, while both ϕ and t diverge. Then, making use of the ansatz $I = -Et + J\phi + W(r,\theta)$, and noting that local change of variable $\chi = \phi - \Omega t$, with $\Omega = \omega(r_H)$, transforms E into $E - \Omega J$, and considering only particles on geodesics with $\theta = \theta_0$ constant, one arrives at a related formal near-horizon static metric. Thus, the general formula (2.22) gives

$$\Gamma \equiv \mathrm{e}^{-2\,\mathrm{Im}\,I} = \exp\left(-\frac{4\pi(E - \Omega J)}{\sqrt{A'(r_H,\theta_0)B'(r_H,\theta_0)}}\right). \tag{3.15}$$

The dependence on θ_0 is only apparent, in fact an explicit calculation leads to

$$T_H = \frac{3r_H^4 + (l^2 + a^2)r_H^2 - a^2 l^2}{4\pi r_H l\left(r_H^2 + a^2\right)}, \tag{3.16}$$

and this result is in agreement with that obtained by means of surface gravity evaluated at the horizon.

As the second example, let us consider the $D = (4 + n)$-dimensional rotating uncharged BH [7], having only one non-zero angular moment. This black hole solution may be phenomenologically relevant in the possible BHs production at colliders within the brane-world scenario; here n denotes the number of extra dimensions. In fact, the colliding partons are supposed to propagate on an infinitely-thin brane and therefore they have a non-zero impact parameter only on a 2-dimensional plane along the brane. As a consequence, only one non-zero angular parameter about an axis in the brane is present. The associated metric induced on the brane reads [8]

$$\begin{aligned}\mathrm{d}s^2 = {} & -\left(1 - \frac{\mu}{\rho^2 r^{n-1}}\right)\mathrm{d}t^2 - \frac{2a\mu\sin^2\theta}{\rho^2 r^{n-1}}\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\rho^2}{\Delta}\,\mathrm{d}r^2 \\ & + \rho^2\,\mathrm{d}\theta^2 + \left(r^2 + a^2 + \frac{a^2\mu\sin^2\theta}{\rho^2 r^{n-1}}\right)\sin^2\theta\,\mathrm{d}\varphi^2,\end{aligned}$$

where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}}, \qquad \rho^2 = r^2 + a^2\cos^2\theta,$$

with the mass and angular momentum (transverse to the (r,φ)-plane) of the black hole given by

$$M_{BH} = \frac{(n+2)A_{n+2}}{16\pi G}\,\mu, \qquad J = \frac{2}{n+2}\,M_{BH}\,a,$$

a being the angular momentum per unit mass, G the $(4+n)$-dimensional Newton constant, and A_{n+2} the area of a $(n+2)$-dimensional unit sphere given by $A_{n+2} = 2\pi^{(n+3)/2}/\,\Gamma[(n+3)/2]$.

Again the formalism requires the ADM form of the metric, with the knowledge of the lapse function $N^2 = -A(r,\theta)$, and the function $B(r,\theta) = g^{rr}$. The metric in the ADM form reads

$$\mathrm{d}s^2 = -\frac{\rho^2\Delta}{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}\,\mathrm{d}t^2 + \frac{\rho^2}{\Delta}\,\mathrm{d}r^2 + \rho^2\,\mathrm{d}\theta^2 + \left(r^2 + a^2 + \frac{a^2\mu\sin^2\theta}{\rho^2 r^{n-1}}\right)\sin^2\theta(\mathrm{d}\varphi - \omega\,\mathrm{d}t)^2, \tag{3.17}$$

where

$$\omega = \frac{a(r^2+a^2-\Delta)}{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}. \tag{3.18}$$

Thus

$$A(r,\theta) = \frac{\rho^2\Delta}{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}. \tag{3.19}$$

As usual, the black hole horizon is defined by $g^{rr} = 0$, namely $\Delta(r_H) = 0$.

Within the near-horizon approximation, one can repeat all the previous steps and the general formula (2.22) gives again

$$\Gamma \equiv \mathrm{e}^{-2\,\mathrm{Im}\,I} = \exp\left(-\frac{4\pi(E - \Omega J)}{\sqrt{A'(r_H,\theta_0)B'(r_H,\theta_0)}}\right). \tag{3.20}$$

The dependence on the fixed angle θ_0 is again apparent and the Hawking temperature reads

$$T_H = \frac{1}{4\pi r_H}\frac{(n+1)r_H^2 + (n-1)a^2}{r_H^2 + a^2}.$$

Note the dependence on the number of extra spatial dimensions n.

We conclude this section with the following remark. In all rotating cases, one must have $E - \Omega J > 0$. This can be easily proved as follows: the energy and angular momentum of a particle with 4-momentum p^a are $E = -p^a K_a$ and $J = p^a\tilde{K}_a$, respectively, where $K = \partial_t$ and $\tilde{K} = \partial_\phi$ is the rotational Killing field. But the Killing field which is timelike everywhere (including the ergosphere) is not K^a, but is instead $\chi = K + \Omega\tilde{K}$. Hence a particle (including those with negative energy inside the ergosphere) can escape to infinity if and only if $p_a\chi^a < 0$, which gives the wanted inequality

$$p^a(K_a + \Omega\tilde{K}_a) = -E + \Omega J < 0. \tag{3.21}$$

At the same time, it is violated only in the super-radiant regime, where the Boltzmann distribution must be replaced with the full Planck distribution, and thus it is outside the validity of the semi-classical method.

4. Conclusions

The tunnelling method has been reformulated in the case of an arbitrary static black hole solution and restricted only to the leading term, namely neglecting the back reaction on the black hole geometry. The inclusion of back reaction effects can be done and gives rise to sub-leading correction in E [2, 9]. The novelty of our approach has mainly been consisted of the covariant treatment of the horizon singularity, through the use of spatial proper distance. As a result, we have been able to derive the correct Hawking temperature, working in the static Schwarzschild gauge.

The approach can also be extended to extremal cases (BH solutions having the horizon at infinite proper spatial distance) obtaining vanishing Hawking temperature [6].

We have also considered rotating cases in the Boyer–Lindquist gauge, in particular the $(4 + n)$-dimensional rotating asymptotically flat BH with only one non-zero angular momentum. Again, making use of the near-horizon approximation, the tunnelling rate in the leading approximation has been derived and, as a consequence, an expression of the Hawking temperatures in agreement with the values computed by means of evaluation of the surface gravity at horizon, has been obtained. The approach can easily be extended to generic D-dimensional rotating BH solutions that have recently been found [10].

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